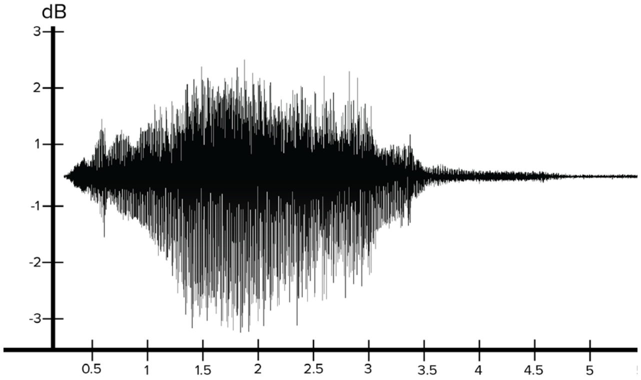
The Mathematics Behind Sound

OVERVIEW



ESSENTIAL QUESTION

What are the mathematical variables that give a sound its particular quality, and how can these variables be measured, visualized, and calculated?

OVERVIEW

In this lesson, students examine the mathematics behind sound by identifying four mathematical variables that define the particular quality of a sound by closely analyzing the sound of four different musical instruments. Students use mathematics to compare and contrast the amplitude, envelope, frequency, and spectrum of each of the four sounds, thereby coming to a better understanding of what physical phenomena contribute to an instrument's unique sound.

Scientifically speaking, sound is nothing more than the vibratory movement of air molecules. But the variations in how such vibrations occur are nearly limitless, creating every sound imaginable, and all the world's musical traditions. Determining and calculating these variables allow someone to understand the mathematics behind sound.

Many of the variables which contribute to a sound's particular quality are measurable thanks to the work of scientists and mathematicians who have created systems that allow us to calculate sound in various ways. These variables include:

Amplitude measures the volume of a sound. In musical terms, it is often known as **dynamics**. Amplitude is calculated in **decibels**, a logarithmic scale, and measured over time with a **level meter.**

Envelope defines how the amplitude of a sound changes over time. In musical terms, envelope is often described as **articulation**. It is measured in decibels in time (usually milliseconds), and is often visually represented and analyzed as a **waveform**.

Frequency defines how high or low a sound is. It is called **pitch** in musical terminology, and is measured in **hertz**. The frequency of a sound can be calculated by dividing the rate of the compressions and rarefactions by the length of a soundwave. An **oscilloscope** is an electronic device that is often used to measure and visualize a sound's frequency.

Spectrum represents how many different frequencies a sound produces. Most of the instruments and objects that create sound vibrate in multiple ways, creating not a single frequency, but a spectrum of frequencies. The various frequencies a sound produces can be measured in decibels, creating a **spectrogram** of the sound. A **spectrometer** is often used to measure the frequency spectrum of a sound over time.

Comprehending and being able to calculate the mathematics behind sound—including amplitude, envelope, frequency, and spectrum—not only leads to a deeper understanding into the physical nature of sound, but is also essential knowledge for musicians and audio engineers who seek to express themselves through sound.

OBJECTIVES

Upon completion of this lesson, students will:

1. KNOW (KNOWLEDGE):

- The mathematics behind sound
- Sound as the perceived result of the compression and rarefaction of air molecules
- Amplitude as a measurement of a sound's volume
- Envelope as a measurement of a sound's amplitude over time
- Frequency as a measurement of a sound's "highness" or "lowness"
- Spectrum as a measurement of a sound's frequency spectrum
- Decibels and hertz as a unit of measuring sound's qualities
- Wavelength and wave speed as attributes of a sound wave determining its frequency
- The crest, trough, and period of a sound wave

2. MASTERY OBJECTIVE:

- Students will be able to identify, calculate, and visually represent the amplitude, envelope, frequency, and spectrum of a sound by closely analyzing the performance of four musical instruments.

ACTIVITIES

MATERIALS NEEDED FOR THIS LESSON:

1. Devices that allow students to access the internet.

PREPARATION:

1. Before starting the Mathematics Behind Sound lesson, it may be helpful to work through (or review) **The Science of Sound** lesson as a class before beginning this lesson, as it introduces some of the terminology and basic concepts used in this lesson.

MOTIVATIONAL ACTIVITY

1. Inform students that they will be investigating the mathematics behind sound in class. They will begin by watching a video of musicians playing a single note on four different instruments. Direct students to make notes describing the best they can how each of the instruments sound.

2. Show students **Clip 1, A Single Note Played on Four Instruments**. Then ask students:

    - What were the instruments you saw in the video? If you can't identify what they are, how would you describe them?
    - Can you describe how some of these instruments sounded, using your own words?
    - How would you compare the sound of one instrument with another?
    - Would it be possible to "measure" the characteristics of these sounds in order to better describe and analyze them?
    - What variables of sound might be able to be measured?

3. Make a list of student responses to the last question on the board (variables mentioned should include the volume, pitch/frequency, length, and "texture" of a sound).

PROCEDURE

1. Inform students that they will be examining the mathematics behind sound by investigating four ways sound can be mathematically measured and shaped: **amplitude, envelope, frequency, and spectrum**. Explain that to better understand these characteristics of sound, they will be comparing and contrasting the four instruments featured in the clip.

2. Pass out to each student **Handout - Variables of Sound Graphic Organizer**. Explain to students that throughout the class they will be filling out the chart (an example of a filled in chart can be found **on Handout - Variables of Sound Graphic Organizer (Teacher's Guide)**.)

3. Show **Image 1, Sound Wave**. Before analyzing the instruments, remind students that sound operates as a **longitudinal wave**—it occurs when a vibrating object displaces air molecules surrounding it, which expand and contract in a regular wave-like motion. When air molecules are close together it is called **compression**. When they are far apart, it is called **rarefaction**. While sound waves are created through compressions and rarefactions, they are represented visually as **transverse** waves, which move up and down like ocean waves. When looked at as a transverse wave, compressions are show by positive numbers on the y-axis, and rarefactions are represented as negative numbers on the y-axis (the x-axis represents time). Representing sound waves in this way makes them easier to analyze. (Note to teacher: these aspects of sound are covered in detail in the **Science of Sound** lesson.) Explain to students that measuring the variables of this sound wave is one way to understand the mathematics behind sound.

4. With **Image 1, Sound Wave** still displayed, inform students that the first variable of sound they will examine is amplitude, which measures the volume of a sound. Pass out **Handout - Amplitude** to students (see **Handout - Amplitude (Teacher's Guide)** for a filled-in version of the handout). Read the introduction of the handout aloud as a class, and remind students to fill out **Handout - Variables of Sound Graphic Organizer** with the pertinent information. Ask students:

    - How would you describe the difference in decibels between these instruments, and how many times louder they are between each other? (Students should notice that, while the instruments only vary by a few decibels, they differ greatly on how much louder they are to the quietest perceptible sound. This reflects the logarithmic scale of decibels).

5. Explain to students that amplitude can only measure the loudness of a sound at a singular moment of time—but it is very common for sounds to change in amplitude over time. These changes in amplitude are known as a sound's **envelope**.

6. Play **Clip 1, A Single Note on Four Instruments** again, and ask students to pay attention to the changes in amplitude that occur with each sound being produced in the video. Ask students:

    - How would you describe how the cymbal changes in amplitude over time?
    - How would you describe how the piano changes in amplitude over time?
    - How would you describe how the saxophone changes in amplitude over time?
    - How would you describe how the bass guitar changes in amplitude over time?

7. Pass out to students **Handout - Envelope** (see **Handout - Envelope (Teacher's Guide)** for a filled-in version of the handout). Read the introduction of the handout aloud as a class, and remind students to fill out **Handout - Variables of Sound Graphic Organizer** with the pertinent information. After filling out the handout, ask students:

    - How would you describe the attack, sustain, and decay of each sound?
    - How might the way an instrument is constructed and played contribute to the instrument's envelope?

8. **(Note: If this lesson is broken into two class sessions, this is an ideal place to end the first session.)** Inform students that the next variable they will be looking at to understand the mathematics behind sound is called **frequency**. Individually or in groups, ask students to go to the **Sound Wave Tech Tool** on their devices. Direct students to make sure the Tech Tool is set to "Sine" at the bottom, and then experiment with the tool by selecting the numbers 1-8. Ask students:

    - How does the sound differ when selecting different numbers?
    - What is the relationship between the numbers, the sound wave graph, and the resulting sound?
    - How many peaks are present in the graph when the number 1 is pushed? How many peaks are present when the number 8 is pushed? What might these peaks represent?

9. Pass out **Handout - Frequency** to students (see **Handout - Frequency (Teacher's Guide)** for a filled-in version of the handout). Teacher's guide here).

Read the introduction of the handout aloud as a class, and remind students to fill out **Handout - Variables of Sound Graphic Organizer** with the pertinent information. Either in groups or individually, have students complete the questions on the handout, and share their answers. Then ask students:

- How would you describe the relationship between the frequency and the wavelength, based upon your answers?

10. Play **Clip 1, A Single Note on Four Instruments** again. After each demonstration, pause the clip and ask students to try to sing or hum the frequency being created by the instrument. Then ask students:

    - Did you find it easier to match the frequency of some instruments over others?
    - Which instruments did you feel were easier to match the frequency? Which were harder?
    - Scientifically speaking, why might some instruments make a clearer frequency than others?

11. Explain to students that in the previous handout they were calculating the fundamental frequencies of the instruments, or the frequencies that the instruments produce most strongly. In reality, most sounds contain multiple vibrating frequencies. The amount and amplitude of different frequencies a sound creates is known as its **sound spectrum** or **spectrum**. Spectrum is the final variable they will consider as they explore the mathematics behind sound.

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13. Pass out **Handout - Spectrum** to students (see **Handout - Spectrum (Teacher's Guide)** to see a filled-in version of the handout.) Read aloud as a class the first paragraph of the handout, and remind students to fill out **Handout - Variables of Sound Graphic Organizer** with the pertinent information. After students complete the handout individually or as a group, discuss the answers to the word problems. Ask students:

    - How do the graphs you created represent the sounds of individual instruments? For example, are the lower frequencies more prominent in the lower sounding instruments?

SUMMARY ACTIVITY

1. Divide students into groups to collectively go over **Handout - Variables of Sound Graphic Organizer**. Using their graphic organizer for support, ask students to turn to their partner and explain briefly what sound is, how it operates and the mathematics behind sound.

2. Inform students that they will conclude the class by examining a real-world application of the calculations performed in class. **Show Clip 2, Sonifying Brain Waves**. Then pass out **Handout - Sonifying Brain Waves Activity**. If desired, students can work on the activity while you play **Clip 3, Background Music for Sonifying Brainwaves Activity**, which was created from Mickey Hart's brain waves.

3. Have students or student groups share their responses with the class.

EXTENSION ACTIVITIES

1. Watch this video of sound engineer and producer Stella Gotshtein describing equalization (found at https://youtu.be/I6ZF_NHvqzU), then write a brief reflection considering what acoustic aspect discussed in the lesson relates to equalization, and why equalization is an essential tool to mixing music. Consider why the mathematics behind sound might be important for a sound engineer to understand.

2. Experiment with the interactive Soundwave and Spectrogram tools at the Chrome music lab (found at https://musiclab.chromeexperiments.com/Experiments), then write a brief paragraph explaining what scientific or mathematical principle each tool is based upon.

STANDARDS

COMMON CORE STATE STANDARDS

The Number System

CCSS.MATH.CONTENT.6.NS.C.6.C: Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

Expressions and Equations

CCSS.Math.Content.6.EE.A.2: Write, read, and evaluate expressions in which letters stand for numbers.

- CCSS.Math.Content.6.EE.A.2.c: Evaluate expressions at specific values of their variables.Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

Statistics and Probability

CCSS.Math.Content.6.SP.B.5: Summarize numerical data sets in relation to their context, such as by:

- CCSS.Math.Content.6.SP.B.5.c: Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

College and Career Readiness Reading Anchor Standards for Grades 6-12 for English Language Arts

Reading 1: Read closely to determine what the text says explicitly and to make logical inferences from it; cite specific textual evidence when writing or speaking to support conclusions drawn from the text.

Craft and Structure 4: Interpret words and phrases as they are used in a text, including determining technical, connotative, and figurative meanings, and analyze how specific word choices shape meaning or tone.

Integration of Knowledge and Ideas 7: Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.

Range of Reading and Level of Text Complexity 10: Read and comprehend complex literary and informational texts independently and proficiently.

College and Career Readiness Anchor Standards for Writing for Grades 6-12

Text Types and Purposes 2: Write informative/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.

College and Career Readiness Anchor Standards for Speaking and Listening for Grades 6-12

Comprehension & Collaboration 1: Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.

Comprehension & Collaboration 2: Integrate and evaluate information presented in diverse media and formats, including visually, quantitatively, and orally.

Presentation of Knowledge 4: Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.

College and Career Readiness Anchor Standards for Language for Grades 6-12

Language 1: Demonstrate command of the conventions of standard English grammar and usage when writing or speaking.

Vocabulary Acquisition and Use 4: Determine or clarify the meaning of unknown and multiple-meaning words and phrases by using context clues, analyzing meaningful word parts, and consulting general and specialized reference materials, as appropriate.

Vocabulary Acquisition and Use 6: Acquire and use accurately a range of general academic and domain-specific words and phrases sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when encountering an unknown term important to comprehension or expression.

RESOURCES

VIDEO RESOURCES

- A Single Note Played on Four Instruments
- Sonifying Brain Waves
- Background Music for Sonifying Brain Waves Activity

HANDOUTS

- Handout - Variables of Sound Graphic Organizer
- Handout - Variables of Sound Graphic Organizer (Teacher's Guide)
- Handout - Amplitude
- Handout - Amplitude (Teacher's Guide)
- Handout - Envelope
- Handout - Envelope (Teacher's Guide)
- Handout - Frequency
- Handout - Frequency (Teacher's Guide)
- Handout - Spectrum
- Handout - Spectrum (Teacher's Guide)
- Handout - Sonifying Brain Waves Activity
- Handout - Sonifying Brain Waves Activity (Teacher's Guide)