Musical Ratios

OVERVIEW

ESSENTIAL QUESTION

What role do ratios play in Western musical concepts of rhythm and harmony?

OVERVIEW

In this lesson, students simplify ratios and find equivalent ratios using proportions as they investigate rhythm and harmony in Western music, recreate the 7-note Pythagorean scale, and calculate ratios between musical intervals.

Musicians and composers could be thought of as chefs. They gather a collection of sonic ingredients—whether from instruments, electronics, or nature—and mix them to create something new and unique. And just like cooking, the creation of music is greatly aided by shared concepts and standardized measurements.

Such musical conceptions and measurements vary across the globe, depending upon the culture and history of the given region. The system often used for Western classical and popular music has developed over hundreds of years, starting when the Holy Roman Emperor Charlemagne initiated a project of writing and standardizing notation for religious chants.

Two of the primary components of the Western musical system are rhythm and harmony. **Rhythm** defines how often sounds are played in a given period of time. **Harmony** speaks to the relationships between the pitches or frequencies of the notes played.

Both rhythm and harmony are based on mathematical ratios.

For rhythm, these ratios are derived rather simply: they are based on the number of notes or sounds played in relation to the beat, or underlying pulse, of a piece of music. For instance, one note could be played in the span of four, three, two, or one beat(s). Conversely, one to eight or more notes could be evenly played in the span of a single beat. Often in music, multiple rhythms occur simultaneously, and the ratios between these contrasting rhythms often inform how complex a piece of music sounds.

The use of ratios in Western music harmony is more complex, going at least as far back as Pythagoras' experiments with a monochord, a single stringed instrument, in the 6th century B.C.E. In his experiments, Pythagoras discovered that plucking a string at certain ratios to its total length created sounds considered pleasant by the ancient Greeks.

From these experiments, he derived a collection of seven notes for creating music. These notes were not based upon specific frequencies, but rather derived from specific ratios in relation to a root pitch. In other words, it was the ratios between the notes that created the Pythagorean tuning system - it was only much later that the notes in the Western musical system were set to specific frequencies, and expanded into twelve total notes. Even today, Pythagoras' seven-note based system is regularly used, and referred to as a **scale.**

OBJECTIVES

Upon completion of this lesson, students will:

1. KNOW (KNOWLEDGE):

- Ratios are used to make comparisons between two things
- How to find equivalent ratios using proportions
- The definition of "rhythm," "interval," and "harmony" in Western music theory, and how these components of music are derived from ratios
- The ways the ratios between two pitches determine their sonic characteristics
- The mathematics behind the Pythagorean 7-note scale

2. MASTERY OBJECTIVE:

- Students will be able to simplify ratios and find equivalent ratios using proportion by investigating rhythm in Western music, recreating the Pythagorean Scale and calculating the C scale tuning ratios.

ACTIVITIES

MOTIVATIONAL ACTIVITY

1. Tell students in this lesson they will be discussing how mathematical ratios operate in music. Play **Clip 1, Musical Ratios**. Then ask students:

   - According to the video, why do different objects produce different sounds? How does that lead to the creation of music?
   - According to the video, what do ratios describe? (*The relationship between two things.*)
   - What does a musical ratio described? (*The relationship between two frequencies.*)
   - What is a 2:1 ratio called in music (*An octave.*)
   - Who was one of the early mathematicians interested in ratios and musical ratios? (*Pythagoras.*) What tool did he use to help calculate musical ratios? (*A monochord.*)
   - How would you describe the monochord?

PROCEDURE

1. Tell students that today they are going to be exploring how mathematical ratios can be used to create and describe music.

2. Show **Image 1, Writing Ratios**. Explain to students that ratios are used to compare two things, and can be written in a variety of ways.

3. Explain that ratios can be used to describe, represent, and create music. There are many different musical systems throughout the world which use ratios, and the one that the class will be focusing on is the Western system, which is the basis of Classical and most Popular music.

4. Tell students that one of the simplest ways ratios are used in music is in **rhythm**, or the ways the sounds in a piece of music are organized in time. Rhythm is usually based on the **beat**, or an underlying pulse of a song.

5. Show **Image 2, Rhythm Activity, Part 1**. Tell students that rhythms are built by subdividing the beat. The image above presents some of the most common ways to subdivide the beat.

6. Play **Clip 2, Simple Rhythms**. Tell students they will be using the clip as a guide to clap a similar rhythmic exercise. The teacher will be clapping the beat in a slow, even rhythm (the white numbers in the clip), and the class will be clapping a ratio of that beat, as determined by the ratios listed in the image (the red numbers in the clip.) For instance, for the 4:1 ratio, students will be clapping once for every four beats the teacher will clap. Conversely, in the 1:4 ratio, students will be clapping four times for every single time the teacher claps. Remind students that subdivisions occur evenly, meaning they have the same amount of duration between them. (If helpful, the teacher may also count "1, 2, 3, 4, 1, 2, 3, 4, etc." to ensure each subdivision is even. For example, in the 4:2 rhythm, students should be clapping together on beats 1 & 3.)

7. Show **Image 3, Rhythm Activity, Part 2**. Tell students that while Western music is often built on multiples and factors of 4, this isn't always the case. Musicians could play 3, 5, 7, or 9 notes per beat. Additionally, in music, multiple rhythms are often played at once.

8. Tell students they will be watching a video of two even rhythms occurring at the same time, as outlined in the above image. Encourage students to take notes on each performance, paying particular attention to how "complex" or "busy" the rhythms sound.

9. Play **Clip 3 Clapping Polyrhythms**, then ask students:

- Which performance sounded more rhythmically "complicated" to you? Which sounded more "simple"?
- Is there a relationship between the "complexity" of the rhythm and its ratio? If so, what is the relationship? (**Note: If this lesson is broken into two class sessions, this is an ideal place to end the first session.**)

10. Explain to students that pitch in music also operates according to ratios. Western music is often built on different groups of seven different pitches called "scales." The most simple of these scales was developed by the Ancient Greek Mathematician Pythagoras, using ratios—though similar systems were developed before that in ancient Mesopotamia.

11. Pass out **Handout - Calculating the Pythagorean Scale** to students. Explain that they will be conducting the same sort of calculations Pythagoras conducted to create a seven-note scale. Using the **Calculating the Pythagorean Scale (Teacher's Guide)**, have students work through the handout individually, in teams, or work through it together as a class.

12. Invite students to investigate what the Pythagorean scale sounds like. Pass out **Handout - Harmony and Interval Chart** to each student. As a class, read the introductory paragraph.

13. Pair students up, and ask each student to go to the TeachRock Techtool (http://bit.ly/trsynth) on their personal or a classroom device. Inform students that this tool plays the notes they calculated in the Pythagorean scale. Point out to students that the techtool lists the notes numerically rather than alphabetically, so "C" is "1", "D" is "2," and so on.

14. Inform students they will be using the techtool to observe how pairs of notes in the scale sound when played together. Ask one student in the pair to hold down the "1" (C) button on the techtool, while the other person presses each additional number in sequence (2, 3, 4. . .). As a pair, students should listen closely to the sound of each interval, and write their observations on the chart in **Handout – Harmony and Interval Chart**. Encourage students that there are no "right" answers in describing these intervals. Inform students that the "ratio" portion of the chart and the questions below the chart will be filled in later in the lesson.

15. Once students have completed their chart, explain to them that they will be calculating the ratios between each interval. Pass out to each student **Handout - Calculating Interval Ratios**. Using the **Calculating Interval Ratios (Teacher's Guide),** have students work through the handout individually, in teams, or work through it together as a class.

16. After completing **Handout - Calculating Interval Ratios**, have students return to **Handout - Calculating the Pythagorean Scale**, and enter the mathematical ratios they calculate for each interval in the "ratio" column on the graph. Have students complete the questions below the graph, and then discuss their answers as a class.

SUMMARY ACTIVITY

1. Go through each of the seven rows in the chart on Handout - Calculating Interval Ratios, and with a show of hands, ask students to vote on which of the seven they felt were the most "pleasant" sounding. Then ask students:

   - Do you think there is a relationship between the simplicity of an interval ratio and how "pleasant" it sounds?
   - Based on the results of the vote, would you say that humans across the globe are biologically inclined to find some intervals more appealing than others?

2. Play **Clip 1, Two Melodies** once again. Ask students:

   - Using what you learned about ratios and intervals in class today, why might the two melodies played in this video sound different or evoke a different mood?

3. Pass out to students **Handout - Ratio Word Problems**. Have students work on the word problems on their own to reinforce the skills and concepts explored in this lesson. They can use the Teacher's Guide as needed.

EXTENSION ACTIVITIES

1. Using **Extension Activity - Interpreting the Graphs of Sound Waves**, examine the sine wave graphs of various intervals, and consider how the mathematical ratios of intervals compare to their visual representations.

2. The golden ratio is the irrational number φ (phi), and like pi, it never ends. The climax in songs is often found at roughly the "phi point" (61.8%) of the song, as opposed to the middle or end of the song. Using **Image 2, Golden Ratio Formula**, have students choose a song and calculate this point. (Note that students will be using 61.8%, which rounds the irrational number to the thousandths place). Then ask students to listen to the song and write a response to the "phi moment" of the song using the formula. Can it be considered the "climax" or "high point" of the song?

STANDARDS

COMMON CORE STATE STANDARDS

Ratios and Proportional Relationships

CCSS.Math.Content.7.RP.A.2 Recognize and represent proportional relationships between quantities.

CCSS.Math.Content.7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

College and Career Readiness Reading Anchor Standards for Grades 6-12 for English Language Arts

Reading 1: Read closely to determine what the text says explicitly and to make logical inferences from it; cite specific textual evidence when writing or speaking to support conclusions drawn from the text.

Craft and Structure 4: Interpret words and phrases as they are used in a text, including determining technical, connotative, and figurative meanings, and analyze how specific word choices shape meaning or tone.

Integration of Knowledge and Ideas 7: Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.

Range of Reading and Level of Text Complexity 10: Read and comprehend complex literary and informational texts independently and proficiently.

College and Career Readiness Anchor Standards for Speaking and Listening for Grades 6-12

Comprehension & Collaboration 2: Integrate and evaluate information presented in diverse media and formats, including visually, quantitatively, and orally.

Presentation of Knowledge 4: Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.

College and Career Readiness Anchor Standards for Language for Grades 6-12

Language 1: Demonstrate command of the conventions of standard English grammar and usage when writing or speaking.

Vocabulary Acquisition and Use 4: Determine or clarify the meaning of unknown and multiple-meaning words and phrases by using context clues, analyzing meaningful word parts, and consulting general and specialized reference materials, as appropriate.

Vocabulary Acquisition and Use 6: Acquire and use accurately a range of general academic and domain-specific words and phrases sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when encountering an unknown term important to comprehension or expression.

RESOURCES

VIDEO RESOURCES

- Two Melodies
- Video Title (if not partner video)

HANDOUTS

- Handout - Calculating the Pythagorean Scale
- Handout - Calculating the Pythagorean Scale (Teacher's Guide)
- Handout - Harmony and Interval Chart
- Handout - Calculating Interval Ratios
- Handout - Calculating Interval Ratios (Teacher's Guide)
- Handout - Ratio Word Problems
- Handout - Ratio Word Problems (Teacher's Guide)
- Extension Activity - Interpreting the Graphs of Sound Waves